5-2: Adding and Subtracting Rational Expressions (Book 9.1)

Objectives:

1. **I can simplify a rational expression**
2. **I can add and subtract rational expressions.**

Add the following fractions:

$\frac{2}{2} \cdot \frac{3}{5} + \frac{3}{10} =$

$= \frac{6}{10} + \frac{3}{10}$

$= \frac{9}{10}$

$\frac{1}{4} + \frac{9}{7} =$

What do you need?

COMMON DENOMINATOR

Multiply by 1

Explain 1 Writing Equivalent Rational Expressions

Given a rational expression, there are different ways to write an equivalent rational expression. When common terms are divided out, the result is an equivalent but simplified expression.

Example 1 **Rewrite the expression as indicated.**

(A) Write $\frac{3x}{(x+3)}$ as an equivalent rational expression that has a denominator of $(x+3)(x+5)$.

The expression $\frac{3x}{(x+3)}$ has a denominator of $(x+3)$.

The factor missing from the denominator is $(x+5)$.

Introduce a common factor, $(x+5)$.

$$\frac{3x}{(x+3)} = \frac{3x(x+5)}{(x+3)(x+5)}$$

$\frac{3x}{(x+3)}$ is equivalent to $\frac{3x(x+5)}{(x+3)(x+5)}$.

(B) Simplify the expression $\frac{(x^2+5x+6)}{(x^2+3x+2)(x+3)}$.

Write the expression. $\frac{(x^2+5x+6)}{(x^2+3x+2)(x+3)}$

Factor the numerator and denominator. ______________

Divide out like terms. ______

Your Turn

2. Write $\frac{5}{5x-25}$ as an equivalent expression with a denominator of $(x-5)(x+1)$.

3. Simplify the expression $\frac{(x+x^3)(1-x^2)}{(x^2-x^6)}$.

Given two or more rational expressions, the least common denominator (LCD) is found by factoring each denominator and finding the least common multiple (LCM) of the factors. This technique is useful for the addition and subtraction of expressions with unlike denominators.

Least Common Denominator (LCD) of Rational Expressions

To find the LCD of rational expressions:

1. Factor each denominator completely. Write any repeated factors as powers.
2. List the different factors. If the denominators have common factors, use the highest power of each common factor.

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Example 2 **Find the LCD for each set of rational expressions.**

(A) $\frac{-2}{3x-15}$ and $\frac{6x}{4x+28}$

Factor each denominator completely.

$3x - 15 = 3(x - 5)$

$4x + 28 = 4(x + 7)$

List the different factors.

$3, 4, x - 5, x + 7$

The LCD is $3 \cdot 4(x - 5)(x + 7)$,

or $12(x - 5)(x + 7)$.

$\frac{-2}{3(x-5)}$ $\frac{6x}{4(x+7)}$

LCD: $3 \cdot 4 \cdot (x-5)(x+7)$

$12(x-5)(x+7)$

(B) $\frac{-14}{x^2 - 11x + 24}$ and $\frac{9}{x^2 - 6x + 9}$

Factor each denominator completely.

$x^2 - 11x + 24 =$ $(x-8)(x-3)$

$x^2 - 6x + 9 =$ $(x-3)(x-3)$

List the different factors.

__________ and __________

Taking the highest power of $(x - 3)$,

the LCD is __________.

$\frac{-14}{(x-8)(x-3)}$ $\frac{9}{(x-3)(x-3)}$

LCD: $(x-3)(x-3)(x-8)$

Your Turn

Find the LCD for each set of rational expressions.

5. $\frac{x+6}{8x-24}$ and $\frac{14x}{10x-30}$

$\frac{x+6}{8(x-3)}$ $\frac{14x}{10(x-3)}$

LCD: $8 \cdot 10(x-3)$

$80(x-3)$

6. $\frac{12x}{15x+60} = \frac{5}{x^2+9x+20}$

$\frac{12x}{15(x+4)} = \frac{5}{(x+5)(x+4)}$

LCD: $15(x+4)(x+5)$

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B $\dfrac{2x^2}{x^2 - 5x} - \dfrac{x^2 + 3x - 4}{x^2}$

Factor the denominators.

$$\frac{2x^2}{\boxed{}} - \frac{x^2 + 3x - 4}{x^2}$$

Identify where the expression is not defined. The first expression is undefined when $x = 0$ and when $x = 5$. The second expression is undefined when $x = 0$.

Find a common denominator. The LCM for $x(x - 5)$ and x^2 is __________.

Write the expressions with a common denominator by multiplying both by the appropriate form of 1.

$$\boxed{} \cdot \frac{2x^2}{x(x - 5)} - \frac{x^2 + 3x - 4}{x^2} \cdot \frac{x - 5}{x - 5}$$

Simplify each numerator.

$$= \frac{2x^3}{x^2(x - 5)} - \frac{x^3 - 2x^2 - 19x + 20}{x^2(x - 5)}$$

Subtract.

$$= \frac{\boxed{} + 2x^2 + 19x - 20}{x^2(x - 5)}$$

Since none of the factors of the denominator are factors of the numerator, the expression cannot be further simplified.

Your Turn

Add each pair of expressions, simplifying the result and noting the combined excluded values. Then subtract the second expression from the first, again simplifying the result and noting the combined excluded values.

7. $-x^2$ and $\frac{1}{(1-x^2)}$

$$\frac{(1-x^2)}{(1-x^2)}\frac{-x^2}{1}+\frac{1}{1-x^2}$$

$$=\frac{-x^2+x^4}{1-x^2}+\frac{1}{1-x^2}$$

$$=\frac{-x^2+x^4+1}{1-x^2}$$

$$=\boxed{\frac{x^4-x^2+1}{1-x^2}}\quad \boxed{x\neq 1,-1}$$

8. $\frac{x^2}{(4-x^2)}$ and $\frac{1}{(2-x)}$

$$\frac{x^2}{4-x^2}-\frac{1}{2-x}$$

$$(2)^2\quad(x)^2$$

$$=\frac{x^2}{(2+x)(2-x)}-\frac{1}{(2-x)}\frac{(2+x)}{(2+x)}$$

$$=\frac{x^2}{(2+x)(2-x)}-\frac{(2+x)}{(2-x)(2+x)}$$

$$=\boxed{\frac{x^2-x-2}{(2+x)(2-x)}\quad x\neq -2,2}$$

Add or subtract the following rational expressions

$$\frac{1}{x-3}+\frac{2}{x^2-4x+3}$$

$$=\frac{(x-1)}{(x-1)}\frac{1}{(x-3)}+\frac{2}{(x-3)(x-1)}$$

$x-1+2$

$$=\frac{(x-1)}{(x-1)(x-3)}+\frac{2}{(x-3)(x-1)}$$

$$=\frac{2+x-1}{(x-1)(x-3)}=\boxed{\frac{x+1}{(x-1)(x-3)}\quad x\neq 1,3}$$

$$\frac{3x}{x+5}-\frac{7}{x^2+3x-10}$$

$$\frac{(x-2)}{(x-2)}\frac{3x}{x+5}-\frac{7}{(x+5)(x-2)}$$

$$=\frac{3x(x-2)-7}{(x-2)(x+5)}=\boxed{\frac{3x^2-6x-7}{(x-2)(x+5)}\quad x\neq 2,-5}$$

Add or subtract the following rational expressions

$$\frac{1}{x-3}+\frac{2}{x^2-4x+3}$$

$$\frac{3x}{x+5}-\frac{7}{x^2+3x-10}$$

Add or subtract the following rational expression

$$\frac{3x}{x^2-5x-6}-\frac{2}{x^2+2x+1}$$

Explain 4 Adding and Subtracting with Rational Models

Rational expressions can model real-world phenomena, and can be used to calculate measurements of those phenomena.

Example 4 **Find the sum or difference of the models to solve the problem.**

(A) Two groups have agreed that each will contribute \$2000 for an upcoming trip. Group A has 6 more people than group B. Let x represent the number of people in group A. Write and simplify an expression in terms of x that represents the difference between the number of dollars each person in group A must contribute and the number each person in group B must contribute.

$$\frac{2000}{x} - \frac{2000}{x-6} = \frac{2000(x-6)}{x(x-6)} - \frac{2000x}{(x-6)x}$$

$$= \frac{2000x + 12{,}000 - 2000x}{x(x-6)}$$

$$= \frac{12{,}000}{x(x-6)}$$

(B) A freight train averages 30 miles per hour traveling to its destination with full cars and 40 miles per hour on the return trip with empty cars. Find the total time in terms of d. Use the formula $t = \frac{d}{r}$.

Let d represent the one-way distance.

$$\text{Total time: } \frac{d}{30} + \frac{d}{40} = \frac{d \cdot \boxed{40}}{30 \cdot \boxed{40}} + \frac{d \cdot \boxed{30}}{40 \cdot \boxed{30}}$$

$$= \frac{d \cdot \boxed{40} + d \cdot \boxed{30}}{\boxed{1200}}$$

$$= \left(\frac{\boxed{40+30}}{\boxed{1200}}\right)d \qquad t \approx \frac{70}{1200}d.$$

Your Turn

9. A hiker averages 1.4 miles per hour when walking downhill on a mountain trail and 0.8 miles per hour on the return trip when walking uphill. Find the total time in terms of d. Use the formula $t = \frac{d}{r}$.

$$t = \frac{d}{1.4} + \frac{d}{0.8}$$