5-4 Inverse Functions

Objectives:

-I can find the inverse of a given function graphically and algebraically

-I can analyze the domain of a function and its inverse

- I can verify inverses using composition

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (*x, y*) is the set of all ordered pairs (*y, x*).

Notation:

$$f^{-1}(x)$$

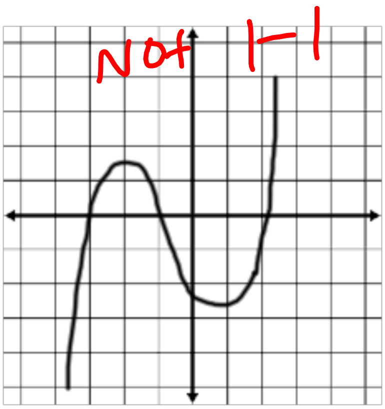
Represents the inverse of the function $f(x)$

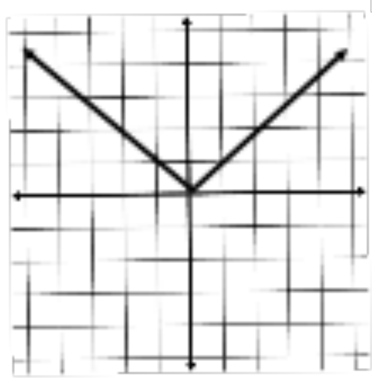
Horizontal-Line Test

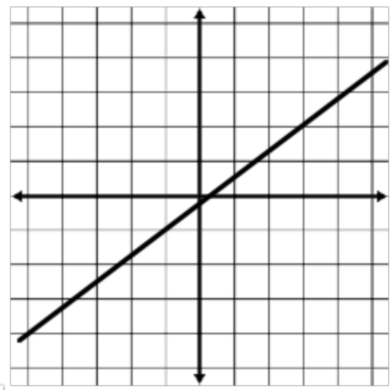
The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

Determine whether the function is one-to-one.





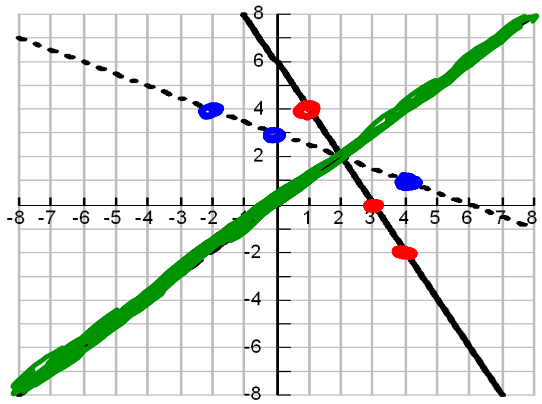


Inverses - graphically

Inverse relations are reflections of each other over the line y = x (identity function)

Property of inverse relations: Suppose *f* & f^{-1} are inverse relations, then *f*(a)=b iff f^{-1}(b)=a

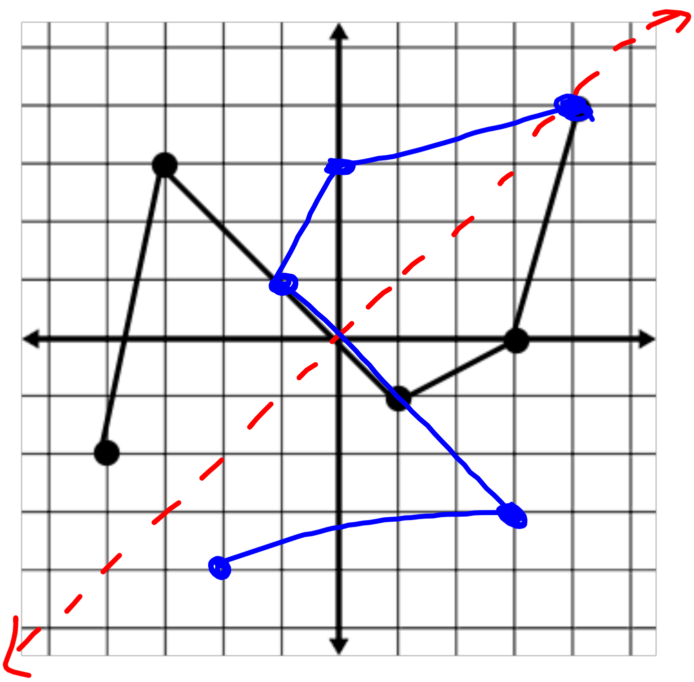
Show $f(x) = 6 - 2x$ and $g(x) = \dfrac{6-x}{2}$
are inverses graphically



f(x): (1,4) (3,0) (4,-2)

g(x): (4,1) (0,3) (-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)



To find the inverse equation of a function

1. Change *f(x)* to *y*.
2. Interchange *x* and *y*
3. Solve for *y*
4. Change new *y* to $f^{-1}(x)$

Find the inverse of each function

$f(x) = x^2 + 1$

$y = x^2 + 1$

$x = y^2 + 1$

$-1 \qquad -1$

$\sqrt{x-1} = \sqrt{y^2}$

$y = \sqrt{x-1}$

$f^{-1}(x) = \sqrt{x-1}$

$g(x) = \dfrac{x+1}{2x+3}$

$(2y+3)x = \dfrac{y+1}{2y+3}(2y+3)$

$(2y+3)x = y+1$

$2xy + 3x = y + 1$

$-3x \qquad -y$

$2xy - y = 1 - 3x$

$\dfrac{y(2x-1)}{2x-1} = \dfrac{1-3x}{2x-1}$

$y = \dfrac{1-3x}{2x-1}$

$g^{-1}(x) = \dfrac{1-3x}{2x-1}$

Find the inverse of each function.

$$h(x) = 2x^3 + 3 \qquad g(x) = \sqrt[3]{x} - 3$$

$x - 3 = 2y^3$

$\sqrt[3]{\frac{x-3}{2}} = y$

$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

or

$h^{-1}(x) = \sqrt[3]{\frac{1}{2}x - \frac{3}{2}}$

$x + 3 = \sqrt[3]{y}$

$g^{-1}(x) = (x+3)^3$

Find the inverse of f(x) and state the domain of $f^{-1}(x)$ including any inherited restrictions

$$f(x) = 2x + 5$$

D $(-\infty, \infty)$
R $(-\infty, \infty)$

$x = 2y + 5$

$x - 5 = 2y$

$y = \frac{x-5}{2}$

$f^{-1}(x) = \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2}$

Find the inverse of f(x) and state the domain of $f^{-1}(x)$ including any inherited restrictions

$$f(x)=\sqrt{2x-3}$$

$$(x)^2=\left(\sqrt{2y-3}\right)^2$$

$$x^2=2y-3$$

$$x^2+3=2y$$

$$\frac{x^2+3}{2}=y \qquad f^{-1}(x)=\frac{x^2+3}{2}$$

Find the inverse of f(x) and state the domain of $f^{-1}(x)$ including any inherited restrictions

$$f(x)=\frac{4x+1}{x-2}$$

$$(y-2)x=\frac{4y+1}{y-2}(y-2)$$

$$x(y-2)=4y+1$$

$$xy-2x=4y+1$$

$$xy-4y=1+2x$$

$$\frac{y(x-4)}{x-4}=\frac{1+2x}{x-4}$$

$$y=\frac{1+2x}{x-4}$$

$$f^{-1}(x)=\frac{2x+1}{x-4}$$

H

We can verify that two functions are inverses of each other by determining if the composition of the two functions are both equal to x.

$$f \circ g = x \qquad g \circ f = x$$

$$f \circ f^{-1} = x \qquad f^{-1} \circ f = x$$

H

Use composition to determine if the following functions are inverses of each other.

$$f(x) = 5x + 1$$

$$g(x) = \frac{x-1}{5}$$

$(f \circ g)(x)$ $f(g(x))$

$f(g(x)) = 5\left(\frac{x-1}{5}\right)+1 = x-1+1 = \boxed{x}$

$g(f(x)) = \frac{5x+1-1}{5} = \frac{5x}{5} = \boxed{x}$

H

Use composition to determine if the following functions are inverses of each other.

$$f(x) = \frac{x-1}{4}$$

$$g(x) = 4x + 1$$